

# Abelian Higgs model with charge conjugate boundary conditions

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The abelian Higgs model is studied on the lattice with charge conjugate boundary conditions. A locally gauge invariant operator for the charged scalar field is constructed and the charged scalar particle mass is calculated in the Coulomb phase of the lattice model. Agreement is found with the mass calculated in Coulomb gauge. The gauge invariant scalar field operator is used to calculate the Higgs boson mass in the Higgs region and to show that the charged particle disappears from the spectrum in the confined regime.

## I. INTRODUCTION

The study of quantum chromodynamics using lattice field theory methods has progressed to the stage where small effects due to electrodynamics have to be considered. However, the description of charged particles on a finite lattice with periodic boundary conditions which are typically used in lattice simulations poses some challenges due to violation of Gauss's law and the gauge dependence of the charged particle propagator. See Refs. [1, 2] for early work and [3, 4] for reviews of recent developments.

Recently, Lucini *et al.* [5] have reconsidered the idea of using charge conjugate boundary conditions [6, 7]. In this setup fields at positions differing by a distance equal to the lattice size are related by charge conjugation. In this way a charged particle on the lattice can have oppositely charged images in neighboring lattice volumes and Gauss's law, which is an obstruction in the case of periodic boundary conditions, can be met. Furthermore, a gauge invariant form for the charged field can be obtained.

In Ref. [5] the formalism for charged fields in a finite volume with charge conjugate boundary conditions is set out and specific examples of operators for lattice QED are constructed. In this work we apply the ideas discussed in [5] to a theory of electrodynamics with scalar fields, namely, the abelian Higgs model [8]. The primary purpose is to illustrate the calculation of the charged particle mass in a consistent gauge invariant way in the Coulomb phase of the lattice model. In addition, using a gauge invariant definition of the charged scalar field the Higgs phenomenon and confinement, which are features of the lattice Higgs model in other regions of the phase diagram [9–11], are demonstrated in a new way.

The general formalism for scalar field electrodynamics with charge conjugate boundary conditions follows the development of Ref. [5] and is given in Sec. II. The specific lattice model used in this work is given in Sec. III. The results of lattice simulations are presented in Sec. IV. In Sec. IV A the scalar model results in the absence of a gauge field for periodic and charge conjugate boundary conditions are compared to show that physics is not affected by boundary conditions. In subsequent subsections of Sec. IV some properties of the model in the Coulomb, Higgs and confined regions are discussed. Sec. V gives a summary.

## II. FORMALISM

### A. General

Consider the Euclidean space action for a complex scalar field  $\phi$  with electrodynamics  $S = S_G + S_\phi$  where

$$S_G = \frac{1}{4} \int d^4x F_{\mu\nu} F_{\mu\nu} \quad (1)$$

and

$$S_\phi = \int d^4x [(D_\mu \phi(x))^* D_\mu \phi(x) + m_c^2 \phi^*(x) \phi(x) + \lambda_c (\phi^*(x) \phi(x))^2] \quad (2)$$

with  $F_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$  and  $D_\mu = \partial_\mu + iqA_\mu(x)$ . The action is invariant under the transformations

$$A_\mu(x) \rightarrow A_\mu(x) - \partial_\mu \alpha(x), \quad (3)$$

and

$$\phi(x) \rightarrow e^{iq\alpha(x)} \phi(x). \quad (4)$$

We consider the theory in a finite cubic spatial volume with length  $L$  on a side. The commonly used boundary conditions are periodic

$$A_\mu(x + L\hat{i}) = A_\mu(x), \quad (5)$$

$$\phi(x + L\hat{i}) = \phi(x) \quad (6)$$

for a shift  $L$  in the  $i$ th direction. However, as discussed in [5], it is advantageous to charge conjugate when making a shift, that is, to apply the conditions

$$A_\mu(x + L\hat{i}) = -A_\mu(x), \quad (7)$$

$$\phi(x + L\hat{i}) = \phi^*(x). \quad (8)$$

These charge conjugate boundary conditions are referred to as  $C^*$  boundary conditions in [5]. In order to preserve the charge conjugate boundary conditions the gauge transformation must also have a particular form. Equation (3) implies that

$$\alpha(x) = \beta(x) + \text{constant}, \quad \beta(x + L\hat{i}) = -\beta(x). \quad (9)$$

Then (4) requires that the constant in (9) should be an integer multiple of  $\pi/q$ . The most general gauge transformation is therefore a combination of a local spatially anti-periodic function and a global factor  $\pm 1$  acting on the scalar field. The global phase symmetry of the action is broken by the boundary conditions from  $U(1)$  to  $\mathbb{Z}_2$ .

The charge conjugate boundary conditions also affect the construction of momentum eigenstates in finite volume [5, 12]. This has implications for the lattice simulations that we carry out. The real part of the scalar field is periodic and a zero momentum field can be constructed by integrating over spatial positions. The mass of the particle associated with the field can then be extracted directly from correlation function of the projected field operator. On the other hand, the imaginary part of the field is antiperiodic and in the lowest momentum eigenstate there is a half unit of momentum  $\pi/L$  associated to each anti-periodic spatial direction. In the lattice simulation correlators of real and imaginary fields have to be treated separately. The correlation function of the imaginary part of the field yields an energy which can be used in a dispersion relation to determine the mass.

### B. Charged field operator

The construction of the charge field operator follows Ref. [5]. The charge  $q$  may be a multiple of some elementary charge  $Q = q/q_{el}$ . Consider the operator

$$\Phi_J(x) = e^{-iq \int d^4y A_\mu(y) J_\mu(y-x)} \phi(x) \quad (10)$$

where  $J_\mu(x)$  satisfies  $\partial_\mu J_\mu(x) = \delta^4(x)$  and  $J_\mu(x + L\hat{i}) = -J_\mu(x)$ . Note that a sign is changed compared to Eq. (3.1) in [5] to be consistent with the gauge transformation (3). Under a global transformation  $A_\mu(x)$  is invariant but

$$\phi(x) \rightarrow e^{iq\alpha(x)} \phi(x) = (-1)^Q \phi(x) \quad (11)$$

so

$$\Phi_J(x) \rightarrow (-1)^Q \Phi_J(x). \quad (12)$$

Using the properties of  $J_\mu(x)$  and Eq. (4) it is easy to verify that  $\Phi_J(x)$  is invariant under a local (anti-periodic) gauge transformation.

Lucini *et al.* [5] give specific examples of functions  $J_\mu(x)$  which yield operators that can be used in a calculation. We adopt two of them for this work. First consider a solution for  $J_\mu(x)$  which has the form

$$J_0(x) = 0, \quad J_i(x) = \delta(x_0) \partial_i \Gamma(\mathbf{x}) \quad (13)$$

where  $\Gamma(\mathbf{x})$  is anti-periodic. Lucini *et al.* [5] give an explicit representation for  $\Gamma$  but we do not need it here. Then the operator (10) takes the form

$$\begin{aligned} \Phi_J(x) &= e^{-iq \int d^3y A_\mu(x_0, \mathbf{y}) \partial_\mu \Gamma(\mathbf{y} - \mathbf{x})} \phi(x), \\ &= e^{iq \int d^3y \partial_i A_i(x_0, \mathbf{y}) \Gamma(\mathbf{y} - \mathbf{x})} \phi(x). \end{aligned} \quad (14)$$

In Coulomb gauge  $\partial_i A_i(x) = 0$ ,  $\Phi_J(x)$  just becomes the gauge fixed scalar field which we will denote as  $\phi_c(x)$ . The correlator of the scalar field in Coulomb gauge yields the gauge invariant mass for the charged particle.

Another solution is

$$J_\mu(x) = \frac{1}{2} \delta_{\mu,i} \text{sgn}(x_i) \prod_{\nu \neq i} \delta(x_\nu). \quad (15)$$

The operator (10) with this choice of  $J_\mu(x)$ , denoted as  $\phi_s(x)$ , takes the form

$$\phi_s(x) = e^{i\frac{q}{2} \int_{-x_i}^0 ds A_i(x+s\hat{i})} \phi(x) e^{-i\frac{q}{2} \int_0^{L-x_i} ds A_i(x+s\hat{i})}. \quad (16)$$

The operator  $\phi_s(x)$  consists of the scalar field with strings emanating in the positive and negative  $i$ th spatial directions. The strings join at the boundary and due to the boundary conditions the operator is invariant under local gauge transformations. This operator is a very convenient one for calculation since it can be constructed easily without gauge fixing.

### III. LATTICE FORMULATION

The lattice version of the abelian Higgs model has been extensively studied, for example, in the pioneering work of Refs. [10, 11, 13, 14]. With the compact form of the lattice gauge field in terms of links  $U_{x,\mu} = e^{iqA_\mu(x)}$  the lattice action  $S = S_G + S_\varphi$  takes the form

$$\begin{aligned} S_G &= -\frac{\beta}{2} \sum_P (U_P + U_P^*), \\ S_\varphi &= -\kappa \sum_{x,\mu} (\varphi^*(x) U_{x,\mu} \varphi(x+\hat{\mu}) + h.c.) \\ &\quad + \sum_x \varphi^*(x) \varphi(x) + \lambda \sum_x (\varphi^*(x) \varphi(x) - 1)^2 \end{aligned} \quad (17)$$

where  $U_P$  is the product of links around the elementary plaquettes and  $\beta = 1/q^2$ . This action is usually used with periodic boundary conditions in all directions. The lattice field and parameters are related to the continuum quantities in (2) by

$$\phi(x) = \varphi(x) \sqrt{\kappa}, \quad \lambda_c = \frac{\lambda}{\kappa^2}, \quad m_c^2 = \frac{1 - 2\lambda - 8\kappa}{\kappa}. \quad (18)$$

With charge conjugate boundary conditions one would like to use the gauge invariant operator (16). As discussed in [5] this is facilitated by introducing a lattice action where the matter field carries two units of charge. Following [5] the scalar QED version of the action is

$$\begin{aligned} S_G &= -2\beta \sum_P (U_P + U_P^*), \\ S_\varphi &= -\kappa \sum_{x,\mu} (\varphi^*(x) (U_{x,\mu})^2 \varphi(x+\hat{\mu}) + h.c.) \\ &\quad + \sum_x \varphi^*(x) \varphi(x) + \lambda \sum_x (\varphi^*(x) \varphi(x) - 1)^2 \end{aligned} \quad (19)$$

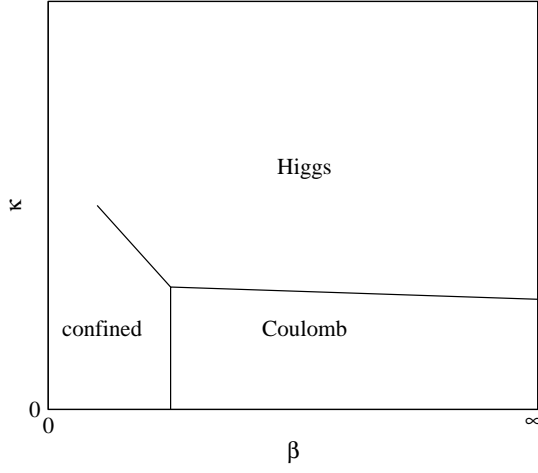


Figure 1: Schematic phase diagram for the lattice abelian Higgs model at fixed  $\lambda$ .

which will be implemented with charge conjugate boundary conditions in all spatial directions and periodic in time. This action is invariant under the local gauge transformations

$$U_{x,u} \rightarrow \Lambda_x U_{x,\mu} \Lambda_{x+\hat{\mu}}^*, \quad (20)$$

$$\varphi(x) \rightarrow \Lambda_x^2 \varphi(x) \quad (21)$$

where the transformation  $\Lambda$  satisfies  $\Lambda_{x+L\hat{i}} = \Lambda_x^*$ .

To investigate the properties of charged field the scalar field after Coulomb gauge fixing  $\varphi_c$  will be used as well as the lattice version of (16) which takes the form

$$\varphi_s(x) = \prod_{s=-x_i}^{-1} U_{x+s\hat{i},i} \varphi(x) \prod_{s=0}^{L-x_i-1} U_{x+s\hat{i},i}^*. \quad (22)$$

#### IV. RESULTS

The phase diagram for the lattice abelian Higgs model [10, 11] at a fixed  $\lambda$  is shown schematically in Fig. 1. One can identify three regions: confined, Higgs and Coulomb. However, the confined and Higgs regimes do not actually correspond to distinct phases as they can be connected by analytic continuation around the transition line that separates the confined and Higgs regions [9]. For  $\lambda \gtrsim 0.1$  the transition line ends at a value of  $\beta$  greater than 0 as shown in the figure. Free charges are expected to exist only in the Coulomb phase [9].

The lattice simulations presented here were carried out on  $16^4$  site lattices using a multi-hit Metropolis updating algorithm. The primary goal is to explore the calculation of the charged particle mass in the Coulomb phase. Evertz *et al.* [13] studied charged particle mass in the abelian Higgs model long ago using an indirect method. To make some contact with this earlier work we choose the same values  $\beta = 2.5$ ,  $\lambda = 3$  for most of the mass cal-

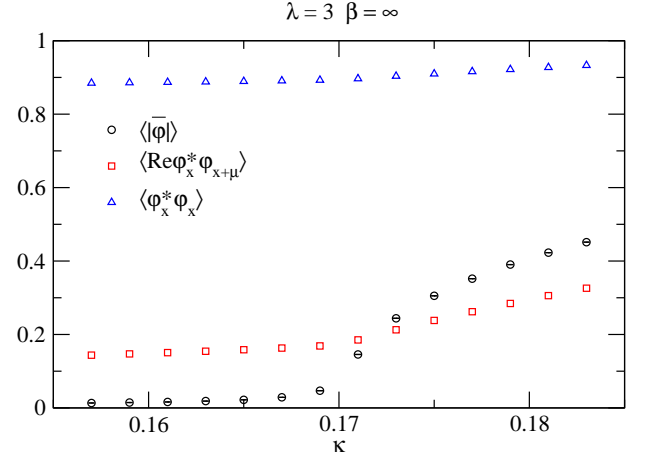


Figure 2: Observables calculated using the scalar field action  $S_\varphi$  with periodic boundary conditions in the absence of a gauge field.

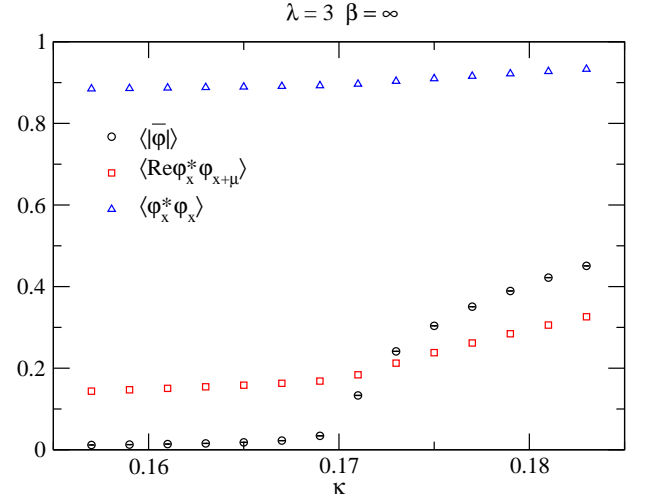


Figure 3: Observables calculated using the scalar field action  $S_\varphi$  with charge conjugate boundary conditions in the absence of a gauge field.

culations. To illustrate the confining feature of the model some calculations at other values of  $\beta$  were also done.

##### A. $\beta = \infty$

In the absence of the gauge field, corresponding to  $\beta = \infty$ , the model reduces to a  $\varphi^4$  theory. As a preliminary step we compare calculations with periodic and charge conjugate spatial boundary conditions (periodic in time) at  $\beta = \infty$ . Ensembles of 32,000 scalar field configurations were used in these calculations.

When  $\kappa$  goes from small to large values there is a transition to a spontaneously broken symmetry phase. To calculate the vacuum expectation value of  $\varphi$ , which would serve as an order parameter, one should introduce

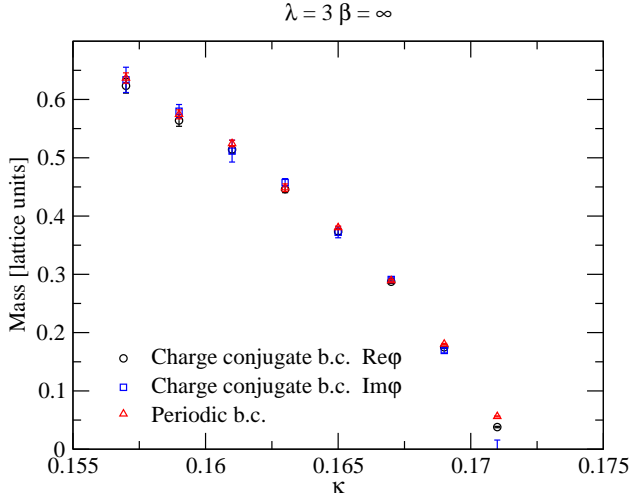


Figure 4: Scalar particle mass in lattice units as a function of  $\kappa$  calculated using the scalar field action in the absence of a gauge field.

a symmetry breaking term with an external field  $\eta$ , for example,  $\eta\varphi$  into the action, calculate  $\langle\varphi\rangle$  and take the thermodynamic and  $\eta \rightarrow 0$  limits. However, there is a simpler procedure without an external field which provides a reasonable estimator for  $\langle\varphi\rangle$  (see Ref. [15]). Consider the field averaged over a single configuration with lattice volume  $V$

$$\bar{\varphi} = \frac{1}{V} \sum_x \varphi(x), \quad (23)$$

and the projection of  $\varphi$  in the direction of  $\bar{\varphi}$

$$\tilde{\varphi}(x) = \frac{\varphi^*(x)\bar{\varphi}}{|\bar{\varphi}|}. \quad (24)$$

Then the expectation value

$$\langle\tilde{\varphi}\rangle = \langle|\bar{\varphi}|\rangle \quad (25)$$

will be used as proxy for  $\langle\varphi\rangle$ . The results for  $\langle|\bar{\varphi}|\rangle$  as a function of  $\kappa$  at  $\lambda = 3$  are shown in Fig. 2 and Fig. 3 for simulations with periodic and charge conjugate spatial boundary conditions respectively. The transition in the vicinity of  $\kappa = 0.17$  is seen clearly. The expectation values of the operators  $\text{Re}(\varphi^*(x)\varphi(x+\hat{\mu}))$  and  $\varphi^*(x)\varphi(x)$  which appear in the action are also shown. Although these are not strictly speaking order parameters their behavior as function of  $\kappa$  can give an indication that the theory undergoes a transition. Simulations with periodic and charge conjugate boundary conditions yield compatible results on our  $16^4$  lattice.

Since the gauge field is absent correlation functions of  $\varphi$  can be used directly to calculate the scalar mass. The results in the symmetric phase are shown in Fig. 4. Recall that with charge conjugate boundary conditions  $\text{Im}\varphi$  is anti-periodic in space so  $\text{Im}\varphi$  is projected to momentum  $(\pi/L)(1,1,1)$ . The energy extracted from the correlator

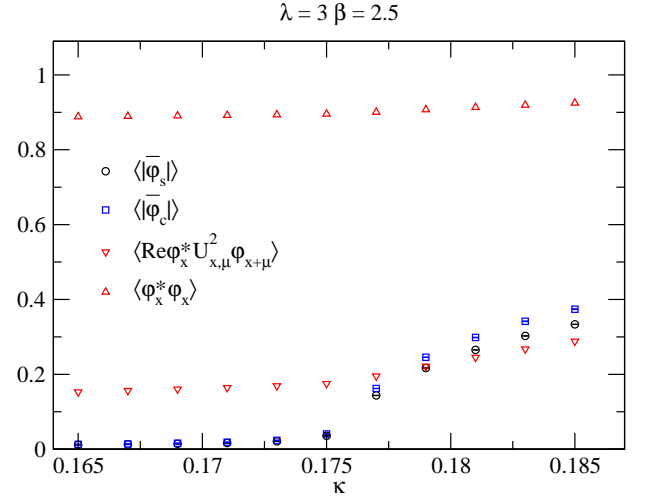


Figure 5: Observables as a function of  $\kappa$  calculated with the action (19) using charge conjugate boundary conditions.

of the momentum projected  $\text{Im}\varphi$  field is converted to a mass using the lattice dispersion relation

$$2 \cosh(E) = m^2 + 8 - 6 \cos(\pi/L). \quad (26)$$

The mass determined this way is consistent with the mass extracted from the zero-momentum correlator of  $\text{Re}\varphi$  as it should be.

## B. Finite $\beta$

Having seen that periodic and charge conjugate boundary conditions give compatible results in the absence of a gauge field we turn in this section to the model at finite  $\beta$ . Figure 5 shows observables calculated at fixed values of  $\beta$  and  $\lambda$  (2.5 and 3 respectively) as a function of  $\kappa$  on a  $16^4$  lattice with the action (19) using charge conjugate boundary conditions. The expectation values of  $|\bar{\varphi}_s|$  and  $|\bar{\varphi}_c|$  show clearly the transition from the Coulomb phase to the Higg regime in the vicinity of  $\kappa$  equal to 0.177. This is very close to the transition point found in Ref. [14] using the action (17) with periodic boundary conditions and with the same values of  $\beta$  and  $\lambda$ . This gives confidence that the physics of the lattice Higgs model used in this work is the same as that of the action (17) used in earlier work.

The primary objective here is to demonstrate the calculation of the charged scalar boson mass in the Coulomb phase. This corresponds to the region that would be relevant for the use of a lattice  $U(1)$  gauge theory in more realistic applications such as electromagnetic corrections to QCD. Correlation functions of four different scalar field operators  $\text{Re}\varphi_s$ ,  $\text{Im}\varphi_s$ ,  $\text{Re}\varphi_c$ ,  $\text{Im}\varphi_c$  were analyzed. Recall that the imaginary parts of the field are anti-periodic in spatial directions so for these fields projection to momentum  $(\pi/L)(1,1,1)$  is carried out and mass is determined

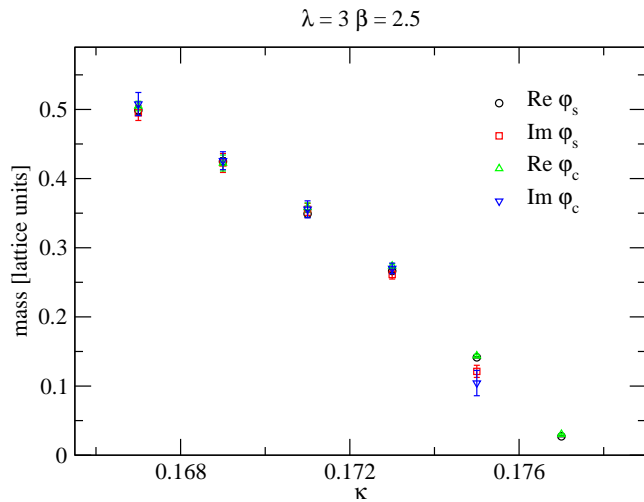


Figure 6: Charged particle mass in lattice units as a function of  $\kappa$  in the Coulomb phase calculated with different operators.

from energy using (26). The real components of the field are projected to zero momentum in the usual way.

For calculations with the gauge invariant operator an ensemble of 32,000 field configurations was used. These were constructed with a multi-hit Metropolis algorithm with 30 sweeps between saved configurations. Since gauge fixing is rather time consuming the Coulomb gauge fixed sample had only 8,000 configurations. The Euclidean time correlation functions were fit with two exponential terms (symmetrized in time). Statistical errors were calculated by a jackknife procedure. The masses in lattice units are plotted in Fig. 6. There is good consistency between different determinations over a range of  $\kappa$  values which provides some confidence that the formulation of the lattice theory presented in [5] can be used effectively to deal with charged particles.

### C. Higgs phase

Since the use of charge conjugate boundary conditions allows for a gauge invariant operator for the scalar field we have a new way to explore the Higgs region. In the standard semi-classical treatment of the Higgs phenomenon the Higgs boson is an elementary field. In contrast, from the nonperturbative perspective of the lattice Higgs model the Higgs boson has been interpolated using a gauge invariant composite operator [14]. For the action (19) the composite Higgs operator takes the form

$$O_H = \text{Re} \sum_i \varphi^*(x) U_{x,i}^2 \varphi(x+i) \quad (27)$$

where the sum is over spatial directions. The construction (22) provides a locally gauge invariant scalar field and it is natural to ask if it also describes the Higgs boson. Correlation functions of  $\text{Re}\varphi_s$  and  $O_H$  (with vacuum expectation values subtracted) were analyzed in the

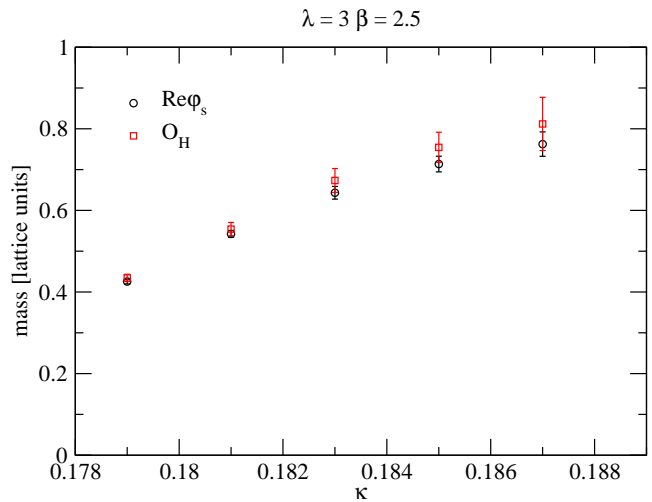


Figure 7: Scalar particle mass in lattice units as a function of  $\kappa$  in the Higgs region calculated with different operators.

Higgs region above  $\kappa = 0.177$ . The mass in lattice units is shown in Fig. 7. The statistical errors are from a jackknife analysis. The masses extracted using the two different fields are consistent over the range of  $\kappa$  values that were investigated. At the upper end of this range the statistical uncertainties are growing so to go to even larger  $\kappa$  would require field ensembles much larger than those used in this study.

The field  $\varphi_s$  is composite but in way that is different from  $O_H$ . It consists of the elementary field  $\varphi$  with a cloud of gauge field fluctuations. It gives a view of the Higgs phenomenon which has some similarity to the semi-classical treatment [8] but without the notion of spontaneous local gauge symmetry breaking which, in the non-perturbative framework, would not be viable [16].

### D. Confinement

At small  $\beta$  compact lattice QED is confining. We explore the transition to the confined regime by calculating at fixed  $\kappa$  and  $\lambda$  and decreasing  $\beta$  starting a point in the Coulomb phase. Figure 8 shows the values of some observables as a function of  $\beta$ . The gauge field plaquette variable  $\text{Re}U_P$  shows the transition from the weak coupling to the strong coupling regime around  $\beta = 0.25$ . The vacuum expectation values of observables involving the  $\varphi$  field are quite insensitive to the value of  $\beta$  and exhibit only small changes in the transition from weak coupling to strong coupling. The mass of the charged scalar extracted from the correlation function of  $\text{Re}\varphi_s$  increases steadily as  $\beta$  is decreased as shown in Fig. 9. Below  $\beta = 0.75$  the correlation function falls very rapidly as function of time so even with an ensemble of 32,000 configurations it was not possible to make an accurate mass determination. Figure 10 shows the correlation function  $\langle \text{Re}\varphi_s(t) \text{Re}\varphi_s(0) \rangle$  at  $\beta = 0.25$ . In this region the correla-

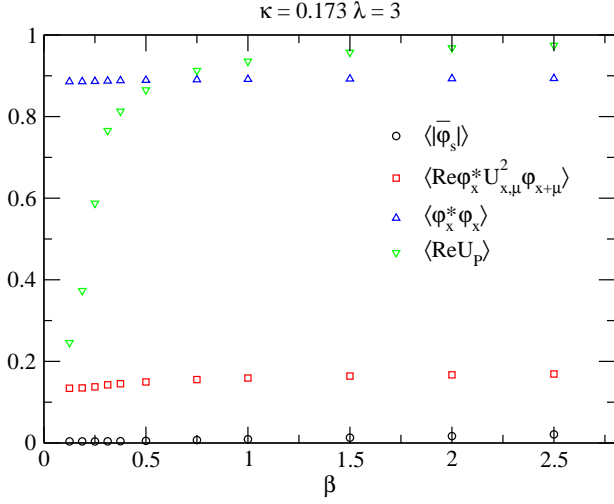


Figure 8: Observables as a function of  $\beta$  calculated with the action (19) with charge conjugate boundary conditions.

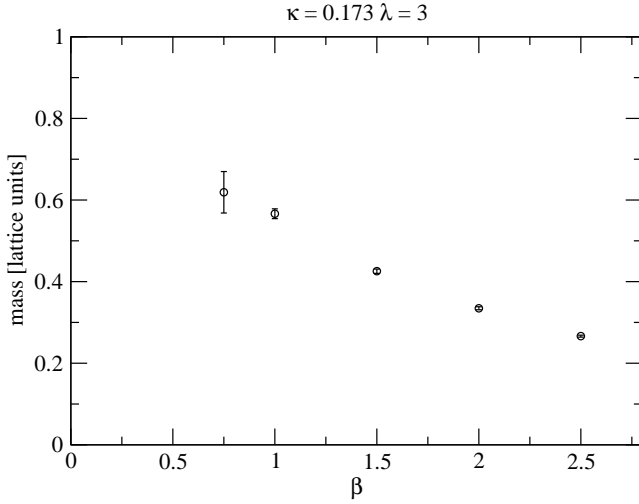


Figure 9: Charged particle mass in lattice units as a function of  $\beta$  calculated using the operator  $\text{Re}\varphi_s$ .

tion function is just noise. The charged scalar field does not propagate. It has disappeared from the spectrum which can be taken as a signature of confinement.

In the strong coupling region the gauge field should also be confined. This can be demonstrated using the photon propagator. For the photon interpolating operator one can use

$$O_p = \text{Im} \sum_{i,j} U_P \quad (28)$$

which is the imaginary part of the gauge field plaquette summed over spatial planes [14]. In the Coulomb phase the photon is expected to be massless [9] so the correlation function should be calculated at a nonzero momentum. We use momentum  $(\pi/L)(1,1,1)$  consistent with our boundary conditions. The energy calculated from

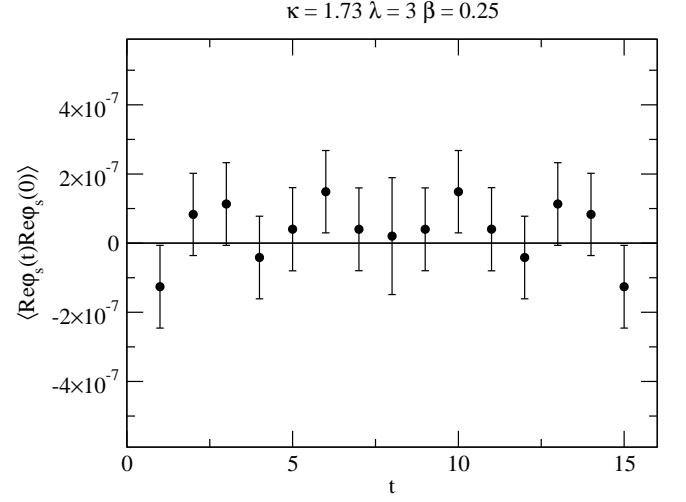


Figure 10: Correlation function  $\langle \text{Re}\varphi_s(t)\text{Re}\varphi_s(0) \rangle$  at  $\beta = 0.25$ .

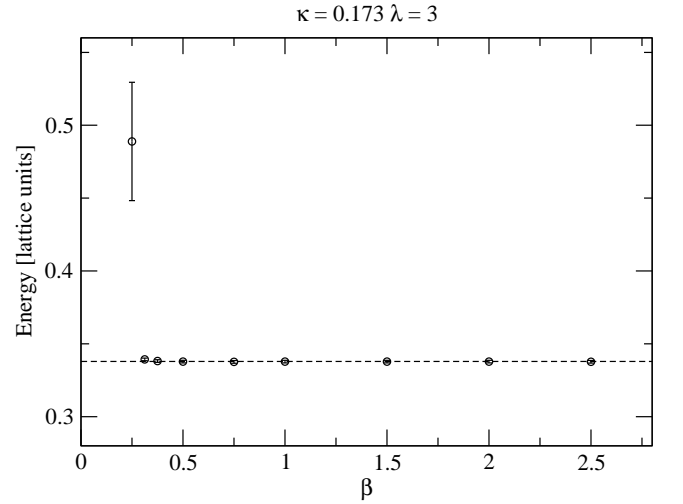


Figure 11: Ground state energy from the correlation function of the operator  $O_p$  (Eq. (28)) as a function of  $\beta$  calculated at momentum  $(\pi/L)(1,1,1)$ . The dashed line shows the energy of a zero mass particle at this momentum from the dispersion relation (26).

the momentum projected correlation function of  $O_p$  is plotted in Fig. (11). The dashed line shows the energy for a zero mass particle calculated using the dispersion relation (26). In the Coulomb phase the gauge field correlator is consistent with the presence of a zero mass photon. Around  $\beta = 0.25$  the mass departs from zero and at smaller values of  $\beta$  the correlator of  $O_p$  is reduced to noise similar to what is seen in Fig. 10 signaling the confinement of the gauge field.

## V. SUMMARY

The use of charge conjugate boundary conditions, as discussed by Lucini *et al.* [5], provides an interesting option for dealing with QED on the lattice. An attractive feature of this formulation is that the mass of the charged field can be determined using a simple gauge invariant procedure. In this paper we have implemented the ideas of [5] in a lattice theory of electrodynamics with scalar fields, the abelian Higgs model.

In Sect. 4.1 the model in the absence of a gauge field ( $\beta = \infty$ ) is compared for charge conjugate and periodic boundary conditions. The results for a variety of observables are compatible. At finite  $\beta$  and other parameters within the perturbative region of the model the charged scalar mass was calculated using both gauge invariant (Eq. (22)) and Coulomb gauge fixed fields. Due to the choice of boundary conditions the imaginary parts of the fields require projection to a non-zero momentum with mass determined using the lattice dispersion relation (26). As shown in Fig. (6) these technically varied

procedures yield compatible charged particle masses.

The gauge invariant field  $\varphi_s$  is also useful for exploring the Higgs model in other regions of the phase diagram. In the Higgs regime the correlator of  $\text{Re}\varphi_s$  gives masses which are compatible with those extracted using the composite scalar operator (27) which has been used in the past to interpolate the Higgs boson. In the strong coupling confining region we showed that the particle associated with field  $\varphi_s$  disappears from the physical spectrum.

In summary, this work demonstrates the efficacy of the formulation of [5] for numerical studies of lattice U(1) gauge theory and encourages further applications.

## Acknowledgments

It is a pleasure to thank C. Itoi for a very helpful discussion. TRIUMF receives federal funding via a contribution agreement with the National Research Council of Canada.

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